Problem 6.15 of Shuler & Kargi. Steady-state operation of a continuous bioreactor.
Instructor: Nam Sun Wang

Conversion of substrate to product by an organism (with negligible biomass production)

$$S \xrightarrow{X} P$$

Monod growth kinetics

$\mu_m := 1$ (h^{-1}) $\quad K_s := 0.01$ (g/L) $\quad \mu(s) := \dfrac{\mu_m \cdot s}{K_s + s}$

Yield coefficient

$Y_s := 0.5$ (g cell/g substrate)

Product formation

$\alpha := 0.4$ (mg product/g cell) $\quad \beta := 0.5$ $(h^{-1}\cdot$mg product/g cell)

Operating condition

Dilution rate $\quad D := 0.8 \cdot \mu_m \quad D = 0.8$ (h^{-1})

Feed substrate concentration $\quad s_0 := 1$ (g/L)

Find steady-states (via analytical solutions)

$$\frac{d}{dt}x = 0 = (\mu(s) - D)\cdot x \longrightarrow \mu(s) = D \longrightarrow s(D) := K_s \cdot \frac{D}{\mu_m - D} \qquad s(D) = 0.04 \text{ (g)}$$

$$\frac{d}{dt}s = 0 = D\cdot(s_0 - s) - \frac{1}{Y_s}\cdot\mu(s)\cdot x \longrightarrow x(D) := Y_s\cdot(s_0 - s(D)) \qquad x(D) = 0.48 \text{ (g)}$$

$$\frac{d}{dt}p = 0 = -D\cdot p + \alpha\cdot\mu(s)\cdot x + \beta\cdot x \longrightarrow p(D) := \left(\alpha + \frac{\beta}{D}\right)\cdot x(D) \qquad p(D) = 0.492 \text{ (mg/L)}$$

Productivity at the given D $\quad D\cdot p(D) = 0.394$ (mg product/h)

Plot steady-states $\quad D := 0.01, 0.02 .. 0.99$