Operating conditions:
\[ s_f := 200 \quad \text{... Feed substrate concentration} \]
\[ D := 0.1 \quad \text{... Dilution rate} \]

Constitutive relations:
\[ \mu_m := 0.3 \quad \text{... maximum specific growth rate} \]
\[ K := 50 \quad \text{... Michaelis-Menten constant} \]
\[ A := 0.004 \quad \text{... constant part of yield coefficient} \]
\[ B := 0.001 \quad \text{... linear part of yield coefficient} \]
\[ \mu(s) := \frac{\mu_m s}{K + s} \quad \text{... Monod specific growth rate} \]
\[ Y(s) := A + B \cdot s \quad \text{... yield coefficient} \]

Dynamic Equations:
\[ \frac{dx}{dt} := (\mu(s) - D) \cdot x \]
\[ \frac{ds}{dt} := D \cdot (s_f - s) - \frac{\mu(s)}{Y(s)} \cdot x \]

Combine individual functions into a vector function suitable for MathCAD's "rkfixed"
\[ ydot(t, y) := \text{vec}(dx/dt, ds/dt) \]

Initial conditions:
\[ x_0 := 0.5 \quad \text{... biomass} \]
\[ s_0 := 30 \quad \text{... substrate} \]

Solve both sets of ODEs from \( t_0 := 0 \) to \( t_f := 100 \) in \( \text{nstep} := 1000 \)
\[ \text{yout} := \text{rkfixed}(ydot, yinitial, t_0, t_f, \text{nstep}, ydot) \]

Extract each variable from columns of "yout"
\[ t := \text{yout}_{0>}, x := \text{yout}_{1<}, s := \text{yout}_{2>} \]

Plots of state variables \( i := 0 \ldots \text{last}(t) \)

A sample of numbers:
\[ x_{\text{nstep}} = 6.523 \quad s_{\text{nstep}} = 22.745 \]

Phase diagram
Solve the same ODEs with the Euler's method in "nstep" steps

\[ i := 0 \ldots nstep \]
\[ h := \frac{t_f - t_0}{nstep} \quad \text{... step size} \]
\[ t_i := i \cdot h \]
\[ x_0 := x_0 \quad \text{... Initial conditions} \]
\[ s_0 := s_0 \]
\[ x_{i+1} := x_i + dxdt(x_i, s_i) \cdot h \]
\[ s_{i+1} := s_i + dsdt(x_i, s_i) \cdot h \]

(Note: coupled equations must be grouped together in a vector.)

For this problem, results from the Runge-Kutta's method and Euler's method are almost indistinguishable.

Extra: Integration by Runge-Kutta's 4th-Order Method without calling "rkfixed"

Slopes for the \(x\) & \(s\) dynamic equations evaluated at 4 different intermediate points ...

\[ \text{ks}_1(x, s) := \text{dsdt}(x, s) \]
\[ \text{ks}_2(x, s) := \text{dsdt}(x + 0.5 \cdot h \cdot \text{ks}_1(x, s), s + 0.5 \cdot h \cdot \text{ks}_1(x, s)) \]
\[ \text{ks}_3(x, s) := \text{dsdt}(x + 0.5 \cdot h \cdot \text{ks}_2(x, s), s + 0.5 \cdot h \cdot \text{ks}_2(x, s)) \]
\[ \text{ks}_4(x, s) := \text{dsdt}(x + h \cdot \text{ks}_3(x, s), s + h \cdot \text{ks}_3(x, s)) \]

Average slope in the interval ...

\[ \text{ks}_\text{ave}(x, s) := \frac{1}{6} (\text{ks}_1(x, s) + 2 \cdot \text{ks}_2(x, s) + 2 \cdot \text{ks}_3(x, s) + \text{ks}_4(x, s)) \]

Compare some numbers:

A sample from Euler's Method:

\[ x_{\text{nstep}} = 6.624 \quad s_{\text{nstep}} = 25.043 \]
\[ t_i := i \cdot h \]
\[ x_0 := x_0 \quad s_0 := s_0 \]

... Initial conditions

A sample from RK Method:
\[ x_{n\text{step}} = 6.523 \quad s_{n\text{step}} = 22.745 \]