Demonstrate simple conversion (Celsius ---> Fahrenheit)

Celsius := 10
Fahrenheit := 1.8*Celsius + 32
Fahrenheit = 50

Demonstrate simple conversion (Fahrenheit ---> Celsius)

Fahrenheit := 10
Celsius := \frac{Fahrenheit - 32}{1.8}
Celsius = -12.222

The following shows three different 2-D plots:
1. a function vs. a range variable
2. a vector vs. an index variable
3. a vector vs. a vector

Demonstrate how to create a table and plot (Celsius ---> Fahrenheit)

C := 0, 10 .. 100  C is treated as an ordinary variable
F(C) := 1.8*C + 32  F is treated as a function

<table>
<thead>
<tr>
<th>C</th>
<th>F(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>68</td>
</tr>
<tr>
<td>30</td>
<td>86</td>
</tr>
<tr>
<td>40</td>
<td>104</td>
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<tr>
<td>50</td>
<td>122</td>
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<tr>
<td>60</td>
<td>140</td>
</tr>
<tr>
<td>70</td>
<td>158</td>
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<tr>
<td>80</td>
<td>176</td>
</tr>
<tr>
<td>90</td>
<td>194</td>
</tr>
<tr>
<td>100</td>
<td>212</td>
</tr>
</tbody>
</table>
Demonstrate how to use index variable and create a table

\[ c := 0, 10 .. 100 \]

C is the index variable; must be integer

\[ F_c := 1.8 \cdot c + 32 \]

F is treated as a vector variable indexed by C, which must be an integer (i.e., 0.1 cannot be used for C).

<table>
<thead>
<tr>
<th>c</th>
<th>( F_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>68</td>
</tr>
<tr>
<td>30</td>
<td>86</td>
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<tr>
<td>40</td>
<td>104</td>
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<tr>
<td>50</td>
<td>122</td>
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<td>140</td>
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<tr>
<td>80</td>
<td>176</td>
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<tr>
<td>90</td>
<td>194</td>
</tr>
<tr>
<td>100</td>
<td>212</td>
</tr>
</tbody>
</table>

Plot a vector vs. an index variable

Plot a vector vs. a vector

Vectorize a function

(Simple \( \sin(x) \) will yield an error message complaining about non-scalar value*. However, some functions such as root, Find, Minerr cannot be vectorized).

\[ x := \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \sin(x) = \begin{bmatrix} 0.841 \\ 0.909 \end{bmatrix} \]
Demonstrate how to solve an algebraic equation of the form $f(x)=0$ with the "root" function.

```plaintext
x := 1  ... Provide the initial guess Different guess may lead to different solution
f(x) := sin(x) - x^2  ... User-defined equation
root(f(x), x) = 0.877  ... Display results directly

or
x := root(f(x), x)  ... Make an assignment first to save the result, then show the results
x = 0.877
```

Make a plot

```plaintext
x := 0, 0.1 .. 2
```

![Plot](image)

The value of a variable used in function definition cannot be changed later, unless that variable is part of the argument. This is because MathCAD processes equations only once and does not go back.

```plaintext
a := 1
f(x) := a + x  \quad f(2) = 3
a := 2  \quad f(2) = 3  \quad <--- Note that there is no change!
```

Demonstrate multivariable equation

```plaintext
f(x, a) := \sin(x) - a \cdot x^2
f(1, 0.5) = 0.341
x := 1
root(f(x, 0.5), x) = 1.404
```
Demonstrate an equation with a vector as argument. (MathCAD Bug: vector or array elements cannot be used as argument to be solved for in Find.)

\[ f(x, a) := a_0 + a_1 \cdot x \]
\[ a_0 := 1 \quad a_1 := 2 \]
\[ f(3, a) = 7 \]
\[ \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = 14 \]

Demonstrate how to create a vector function -- define each element function separately.

Note: In defining the component functions, \( f.1(x) \) is used instead of \( f[1(x) \), which will not work, and note the element of \( f(x) \) is \( f(x)[0 \) instead of \( f[0(x) \) after the component functions are vectorized. Furthermore, each component function cannot be defined separately as \( f(x)[0 \) nor as \( f(x)[i \):

Define component function:

\[ f_0(x) := 1 + x \]
\[ f_1(x) := 2 + 3 \cdot x \]

Combine components:

\[ f(x) := \begin{bmatrix} f_0(x) \\ f_1(x) \end{bmatrix} \]

Vectorized function addressed as a whole:

\[ f(3) = \begin{bmatrix} 4 \\ 11 \end{bmatrix} \]

Vectorized function addressed by components:

\[ f(3)_0 = 4 \]
\[ f(3)_1 = 11 \]

Demonstrate how to create a vector function -- define element functions simultaneously

Define a vector function:

\[ f(x) := \begin{bmatrix} 1 + x \\ 2 + 3 \cdot x \end{bmatrix} \]

Vectorized function addressed as a whole:

\[ f(3) = \begin{bmatrix} 4 \\ 11 \end{bmatrix} \]

Vectorized function addressed by components:

\[ f(3)_0 = 4 \]
\[ f(3)_1 = 11 \]
Demonstrate how to create a vector function of a vector

\[ f(x) := \begin{bmatrix} x_0^2 + x_1^2 - 6 \\ x_0 + x_1 - 2 \end{bmatrix} \quad f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \]

However, a vector function or vector variables cannot be used in "Find".

Demonstrate how to create a function of a matrix.

\[ f(A, x) := A^{-1} \cdot x \quad f\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix} \]

Demonstrate how to create a function of a function.

\[ f(g, x) := g(x) + x \quad g(x) := \begin{bmatrix} x_0 + x_1 \\ x_0 - x_1 \end{bmatrix} \quad f\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \]

Demonstrate how to solve an equation with "Find"

\[ x := 1 \quad \text{... Provide the initial guess} \]

\[ \text{Given} \quad \text{"Given" indicates the start of the equation} \]

\[ \sin(x) = x^2 \quad \text{... given a (set of) equation; note the logical equal sign} \]

\[ \text{Find}(x) = 0.877 \quad \text{... Display results directly} \]

Make a plot

\[ x := 0, 0.1 \ldots 2 \]

\[ \text{A plot} \]

Demonstrate how solve a set of equations with "Find"

\[ x := 0 \quad y := 0 \quad \text{... Provide the initial guess} \]

\[ \text{Given} \quad \text{... Start of the equation} \]

\[ x^2 + y^2 = 6 \quad \text{... 1st equation} \]

\[ x + y = 2 \quad \text{... 2nd equation} \]

\[ \text{Find}(x, y) = \begin{bmatrix} 2.414 \\ -0.414 \end{bmatrix} \quad \text{... A set of answers} \]
Demonstrate how to solve a set of equations with "Find"

\[ x \leq 0 \quad y \leq 0 \quad \text{... Provide the initial guess} \]

Given
\[ \begin{align*}
  x^2 + y^2 &= 6 \quad \text{... 1st equation} \\
  x + y &= 2 \quad \text{... 2nd equation}
\end{align*} \]

\[
\begin{bmatrix}
  x \\
  y
\end{bmatrix} := \text{Find}(x, y) \quad \text{... Save the answer first (to be used later)}
\]

Must be saved explicitly. Otherwise the original initial guess values remain assigned.

\[ x = 2.414 \quad \text{... Display the answer} \]
\[ y = -0.414 \]

Demonstrate how to iterate for a given number of times

\[ x_0 := 1 \quad \text{... Provide the first number} \]
\[ i := 0 \ldots 10 \quad \text{... index variable} \]
\[ x_{i+1} := x_i + 2 \quad \text{... DO loop} \]
\[ a_i := x_i + 1 \]

Table of \( i \) and \( x \)

"x[i]" gives a table; "x=" gives a vector

\[
\begin{array}{ccc}
  i & x_i & a_i \\
  \hline
  0 & 1 & 2 \\
  1 & 3 & 4 \\
  2 & 5 & 6 \\
  3 & 7 & 8 \\
  4 & 9 & 10 \\
  5 & 11 & 12 \\
  6 & 13 & 14 \\
  7 & 15 & 16 \\
  8 & 17 & 18 \\
  9 & 19 & 20 \\
 10 & 21 & 22 \\
\end{array}
\]

\[ x = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ 11 \\ 13 \\ 15 \\ 17 \\ 19 \\ 21 \\ 23 \end{bmatrix} \]
Demonstrate how to solve an equation for a given number of times without an index variable

\[ f(x, a) := \sin(x) - a \cdot x^2 \]

\[ a := 0.0, 0.1 \ldots 1.0 \]

\[ x := 1 \quad \text{... Initial guess} \]

<table>
<thead>
<tr>
<th>(a)</th>
<th>root( (f(x, a), x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>0.3</td>
<td>1.801</td>
</tr>
<tr>
<td>0.4</td>
<td>1.581</td>
</tr>
<tr>
<td>0.5</td>
<td>1.404</td>
</tr>
<tr>
<td>0.6</td>
<td>1.26</td>
</tr>
<tr>
<td>0.7</td>
<td>1.139</td>
</tr>
<tr>
<td>0.8</td>
<td>1.037</td>
</tr>
<tr>
<td>0.9</td>
<td>0.951</td>
</tr>
<tr>
<td>1.0</td>
<td>0.877</td>
</tr>
</tbody>
</table>

Demonstrate how to solve an equation for a given number of times with an index variable

\[ f(x, a) := \sin(x) - a \cdot x^2 \]

\[ N := 10 \]

\[ i := 0 \ldots N \]

dummy := 1

\[ a_i := \frac{i}{N} \]

\[ x_i := \text{root}( f(\text{dummy}, a_i), \text{dummy} ) \]

<table>
<thead>
<tr>
<th>(a_i)</th>
<th>(x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0</td>
</tr>
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<td>0.1</td>
<td>0</td>
</tr>
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<td>0.2</td>
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